DYNAMIC THERMOELASTICITY PROBLEM

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The thermostress state of a two-layer system is investigated by a finite-difference method.

Suppose that a plane-parallel isotropic metallic layer of thickness b_1, occupying the region $\{0 \le z \le b_1, -\infty < x, y < +\infty\}$, is brought into contact along the base $z = b_1$ with another layer of thickness $[b_2 - b_1]$, occupying the region $\{b_1 \le z \le b_2, -\infty < x, y < +\infty\}$, under the assumption that the mechanicothermal characteristics of the layers are different. There is heat transfer through the layer surfaces $z = 0$ and $z = b_2$ with an external medium whose temperature changes at the initial instant from T_0 to T_c, subsequently (for simplicity) remaining constant. At $t = 0$ the temperature of the layer is T_0 and the heating rate is assumed to be zero.

Within the framework of dynamic unbound thermoelasticity theory, the problem of finding the stress state of a two-layer system is posed in dimensionless coordinates in a one-dimensionless formulation as follows [1]: to find in the region

$$D=\{(\tau, \xi),\ 0\leqslant\tau<\tau_1\,(\tau_1<+\infty),\ 0\leqslant\xi\leqslant\xi_1 \cup \xi_1\leqslant\xi\leqslant\xi_2\}$$

the finite solution of the system of equations

$$\frac{\partial^2\Theta_i}{\partial\xi^2}=M_i^2\frac{\partial^2\Theta_i}{\partial\tau^2}+\frac{\partial\Theta_i}{\partial\tau};$$
$$\frac{\partial^2\tilde{W}_i}{\partial\xi^2}-\frac{\partial^2\tilde{W}_i}{\partial\tau^2}=\frac{\partial\Theta_i}{\partial\xi};\quad i=1,\ 2 \tag{1}$$

with the initial conditions ($\tau = 0$)

$$\Theta_i=\frac{\partial\Theta_i}{\partial\tau}=\tilde{W}_i=\frac{\partial\tilde{W}_i}{\partial\tau}=0, \tag{2}$$

boundary conditions

$$\Theta_1|_{\xi=0}=1;\ \Theta_2|_{\xi=\xi_2}=1;\ \tilde{W}_1|_{\xi=0}=\tilde{W}_2|_{\xi=\xi_2}=0, \tag{3}$$

and conditions of ideal thermomechanical contact

$$\Theta_1=\Theta_2;\ \alpha\frac{\partial\Theta_1}{\partial\xi}=\frac{\partial\Theta_2}{\partial\xi}\quad \text{when}\ \xi=\xi_1;$$
$$\gamma\tilde{W}_1=\tilde{W}_2;\quad \beta\frac{\partial\tilde{W}_1}{\partial\xi}-\frac{\partial\tilde{W}_2}{\partial\xi}=\beta\Theta_1-\Theta_2\ \text{when}\ \xi=\xi_1. \tag{4}$$

Here

$$\Theta_i=\frac{T_i-T_0}{T_c-T_0};\quad \tilde{W}_i=\frac{c_{1i}W_i}{s_ia_i(T_c-T_0)};\ \xi=\frac{c_{1i}z}{a_i};\ \tau=\frac{c_{1i}^2t}{a_i};$$
$$M_i=\frac{c_{1i}}{c_{q_i}};\ \xi_1=\frac{c_{1i}b_1}{a_i};\ \xi_2=\frac{c_{1i}W_i}{a_i};\ \alpha=\frac{c_{11}c_1\rho_1}{c_{12}c_2\rho_2};$$
$$\beta=\frac{s_1\rho_1c_{11}^2}{s_2\rho_2c_{12}^2};\quad \gamma=\frac{s_1a_1c_{12}}{s_2a_2c_{11}};\ s_i=\frac{1+\mu_i}{1-\mu_i}\alpha_{T_i};$$

T is the temperature; W, displacement; c_q and c_1, velocities of propagation of the thermal and longitudinal waves; a, μ, α_T, and λ, thermal diffusivity, Poisson's ratio, the linear-expansion coefficient, and the thermal conductivity; ρ, density; c, specific heat.

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TABLE 1. Thermostress State of Contact Boundary

τ	$\Theta_i\cdot 10^{-3}$		$\tilde{W}_i\cdot 10^{-3}$		$\sigma_z^i\cdot 10^{-3}$	
	$i=1$	$i=2$	$i=1$	$i=2$	$i=1$	$i=2$
4,	0,	0,	0,019	0,017	0,005	0,004
5,	0,002	0,002	0,621	0,565	0,136	0,092
6,	0,029	0,029	8,972	8,169	1,671	1,137
7,	0,224	0,224	71,001	64,646	10,967	7,463
8,	1,125	1,125	348,29	317,118	42,34	28,853
9,	4,028	4,028	1141,29	1039,15	99,18	67,492
10,	10,029	10,029	2440,71	2222,27	128,45	87,410

TABLE 2. Thermostress State of Layer Surfaces

τ	$\sigma_z^1(0)$	$\sigma_z^2(20)$	$\sigma_x^1(0)=\sigma_y^1(0)$	$\sigma_x^2(20)=\sigma_y^2(20)$
1	—0,56903	—0,57825	—0,81529	—0,81923
2	—0,14657	—0,18994	—0,63422	—0,65281
3	—0,29027	—0,31894	—0,69581	—0,70810
4	0,47555	0,53798	0,20382	0,23058
5	0,06781	0,13963	0,02906	0,05984
6	—0,02424	—0,01024	—0,01039	—0,00439
7	0,12686	0,12046	0,05437	0,05163
8	0,11149	0,11104	0,04778	0,04759
9	—0,00081	0,00428	—0,00035	0,00183

Constructing the solutions in analytic form leads to complex expressions for the generalized fields [2], and also the displacement and stress fields [3], which poses certain difficulties for their computer realization. Therefore, in investigating the thermostress state of objects it is more sensible, in some cases, to use approximate methods of solving thermoelasticity problems. In the given work, the thermoelastic fields in the two-layer system are determined by a finite-difference method.

Replacing the derivatives with respect to the coordinate and time in Eq. (1) and the conditions (2)-(4) by the corresponding difference relations [4], a finite-difference analog of the postulated problem is obtained. Then, omitting the residual terms which tend to zero when l and h_i tend to zero, the recurrence relations for the displacement take the form

$$\tilde{W}_i^{k,m+1}=\frac{l^2}{h_i^2}[\tilde{W}_i^{k+1,m}+\tilde{W}_i^{k-1,m}]-2\left[\frac{l^2}{h_i^2}-1\right]\tilde{W}_i^{k,m}-\tilde{W}_i^{k,m-1}-\frac{l^2}{h_i}[\Theta_i^{k+1,m}-\Theta_i^{k,m}], \tag{5}$$

where $\Theta_i^{k,m}$ are given by the relations

$$\Theta_i^{k,m+1}=\frac{2(M_i^2h_i^2-l^2)+lh_i^2}{(M_i^2+l)h_i^2}\Theta_i^{k,m}-\frac{M_i^2}{M_i^2+l}\Theta_i^{k,m-1}+\frac{l^2}{(M_i^2+l)h_i^2}\Theta_i^{k+1,m}+$$
$$+\frac{l_i^2}{(M_i^2+l)h_i^2}\Theta_i^{k-1,m};\quad W_i(\xi_k,\tau_m)=\tilde{W}_i^{k,m};\quad \Theta_i^{k,m}=\Theta_i(\xi_k,\tau_m);\ i=1,2. \tag{6}$$

Analogous relations are found for the initial, boundary, and contact conditions. The convergence and stability of the computation process realizing Eqs. (5) and (6) is ensured by consistent choice of the quantization steps over the time l and the coordinate h_i, using the condition

$$l^2/M_i^2h_i^2\leqslant 1.$$

By means of Eq. (5), the displacement at any internal point of the layers can be found from known values of the displacements at the points (ξ_k,τ_{m-1}); (ξ_k,τ_m), (ξ_{k-1},τ_m), (ξ_{k+1},τ_m) and the temperatures at the points (ξ_{k+1},τ_m), (ξ_k,τ_m),

$$k=1,\ 2,\ \ldots,\ \left[\frac{b_i}{h_i}\right];\ m=1,\ 2,\ \ldots,\ \left[\frac{\tau_1}{l}\right].$$

TABLE 3. Temperature Stress in the Vicinity of Contact

τ	ξ			
	7	9	11	12
5	—0,02473	—0,00018	—0,000008	—0,000001
7	—0,46605	—0,02269	—0,00251	—0,000319
9	—1,15087	—0,34638	—0,07834	—0,01949

The calculation of the displacements, organized using a program written in the language MNEMOKOD for an M-6000 computer, proceeds as follows: the columns of the temperature matrix are filled successively at each time level. In filling each column, beginning with the second time level, three regions of time-level formation are isolated using logic units: the boundary, internal, and contact regions. The values for the boundary region are obtained from Eq. (3), those for the internal region from Eq. (6), and those from the contact region by solving Eqs. (4). The first level is formed using Eq. (2). When the calculation of the temperature matrix is complete, it is used in an analogous calculation of the displacements. The level ends when a given value of the time reached. After the calculations are complete, the temperature and displacement matrices calculated are printed out. The stress field is determined from the relation [1]

$$\sigma_z^i = \frac{\partial \tilde{W}_i}{\partial \xi} - \Theta_i;\ \sigma_x^i = \sigma_y^i = -\frac{1-2\mu_i}{1-\mu_i}\Theta_i + \frac{\mu_i}{1-\mu_i}\sigma_z^i,$$

where $\sigma_z^i = \sigma_{zz}^i (1-2\mu_i)/E_i \alpha_{T_i}(T_c - T_0)$; E is Young's modulus.

The temperature, displacement, and stress fields have been calculated for the system aluminum–copper, for which the dimensionless parameters are $M_1^2 = 1.846$; $M_2^2 = 1.29$; $\alpha = 0.8733$; $\beta = 0.6805$; $\gamma = 0.9105$ under the condition that a temperature $\Theta_1 = \Theta_2 = 1$ is maintained for a time $\tau = 3.5$ at the surfaces $\xi = 0$ and $\xi = 20$ of the layers, and then $\Theta_1 = \Theta_2 = 0$. Table 1 shows the temperature, displacement, and stress values at the layer contact point $\xi = 10$, from which it follows that in the given time interval tensile stress appears at the contact boundary and increases; Table 2 shows the values of the temperature stress at the layer surfaces $\xi = 0$ and $\xi = 20$. As is evident from Tables 1 and 2, compressive stress first appears, and is then converted into tensile stress. The stress at the surface layers changes its sign. The temperature–stress values in Table 3, corresponding to conditions such that a temperature $\Theta_2 = 0$ is maintained at the surface $\xi = 20$, show that, in the vicinity of contact, stress of a single sign – compressive stress – appears. Note that the calculations of the dynamic thermoelastic fields made in this case carry over to the case of a multilayer system, and the computation program has been developed with this requirement in mind; for $M_i = 0$, quasistatic temperature fields are calculated. The stability condition for the difference scheme is then [4]

$$l/h_i^2 \leqslant 1/2.$$

LITERATURE CITED

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